Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY
Summer Examination-2018

Subject Name: Mathematics-I

Subject Code: 4SC01MAT1 **Branch: B.Sc. (All)**

Semester: 1 **Date: 23/03/2018** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n =$ ______. **(01)**

(a) 1 (b) e (c) $\log 1$ (d) $\frac{1}{e}$

b) A square matrix A is called skew-symmetric matrix if _________. **(01)**

(a) $A^T = -A$ (b) $A^2 = A$ (c) $A^T = A$ (d) $A^2 = I$

c) n^{th} derivative of $y = \log(3-2x)$ is _________. **(01)**

(a) $\frac{-(2)^n (n-1)!}{(3-2x)^n}$ (b) $\frac{(-1)^n(-2)^n n!}{(3-2x)^n}$ (c) $\frac{2^n n!}{(3-2x)^{n+1}}$ (d) $\frac{(-1)^{n-1}(-2)^n(n-1)!}{(3-2x)^{n+1}}$

d) The one of the Eigen values of the matrix $A=\begin{bmatrix}1 & 2 & 2.5\\ 0 & \sqrt{2} & 3\\ 0 & 0 & -1\end{bmatrix}$ is __________. **(01)**

(a) 1 (b) 2 (c) 3 (d) 0

e) The series $x-\frac{x^3}{3!}+\frac{x^5}{5!}-....$ represent expansion of ________. **(01)**

(a) $\sin x$ (b) $\cos x$ (c) $\sinh x$ (d) $\cosh x$

f) The radius of the sphere $x^2+y^2+z^2-2x+4y-6z=11$ is ________. **(01)**

(a) 5 (b) 2 (c) 4 (d) 6

g) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ is ________. **(01)**

(a) 1 (b) 2 (c) 3 (d) 0

h) $\lim_{x \to 0} \frac{3x}{\tan 3x} =$ _____. **(01)**

(a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0

i) Formula of Lagrange's theorem is ________. **(01)**

(a) $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$ (b) $\frac{f(b)-f(a)}{g(b)-g(a)} = f'(c)$

(c) $\frac{f(b)-f(a)}{b-a} = f'(c)$ (d) $f'(c) = 0$

j) The degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{2}{3}}$ is ________. **(01)**

(a) 1 (b) 2 (c) 3 (d) 6

k) The equation $P(x,y)\,dx + Q(x,y)\,dy = 0$ is exact if ________. **(01)**

(a) $P_x = Q_y$ (b) $P_y = Q_x$ (c) $P_x = -Q_y$ (d) $P_y = -Q_x$

l) The solution of the differential equation $y + px = p^2$ is _______. **(01)**

(a) $y = cx - c^2$ (b) $y = cx + \log c$ (c) $y = c^x + c^2$ (d) none of these

m) What is the nth derivative of a^x? **(01)**

n) A $n \times n$ Non-Homogeneous system of equations $AX = B$ is given. If $\rho(A) = \rho(A:B) = n$ then the system has **(01)**

(a) No solutions (b) Unique solutions

(c) Infinite solution (d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

a) Investigate for what value of λ and μ the equation **(05)**

$x + 2y + z = 8;\ 2x + 2y + 2z = 13;\ 3x + 4y + \lambda z = \mu$ has

i) unique solution, ii) infinite solution and iii) no solution.

b) If $y = \sin^{-1} x$ then prove that $(1-x^2)\,y_{n+2} - (2n+1)\,xy_{n+1} - n^2 y_n = 0$. **(05)**

c) Evaluate: $\lim_{x \to 0} (\cos x)^{\cot x}$ **(04)**

Q-3 Attempt all questions

a) Solve the following system of linear equations **(05)**

$x-y+z=0;\ x+2y+z=0;\ 2x+y+3z=0$

b) Solve: $\left(x^2+y^2\right)dx+2xy\,dy=0;\ \ y(1)=2$ **(05)**

c) Expand $\log\left(1+e^x\right)$ in ascending powers of x as far as the term containing x^4. **(04)**

Q-4 Attempt all questions

a) Evaluate: $\lim\limits_{x\to 0}\dfrac{e^x-1-x}{x\sin x}$ **(05)**

b) Find nth derivative of $\sin x\cdot\sin 2x$. **(05)**

c) Find the centre and radius of the circle is given by **(04)**

$x^2+y^2+z^2+12x-12y-16z+111=0$ and $x+2y+z=17$.

Q-5 Attempt all questions **(14)**

a) Expand $\sin x$ in powers of $\left(x-\dfrac{\pi}{6}\right)$ by Taylor's theorem. **(05)**

b) Find the inverse of the matrix $A=\begin{bmatrix}1 & 2 & 1\\ 2 & 1 & 1\\ 4 & 5 & 1\end{bmatrix}$ by Gauss-Jordan method. **(05)**

c) Find equation of sphere which passes through $(1,-3,4),(1,-5,2),(1,-3,0)$ and whose centre lies on the plane $x+y+z=0$. **(04)**

Q-6 Attempt all questions

a) If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ then verify Cayley-Hamilton theorem and hence find A^4. **(07)**

b) Solve: $\dfrac{dy}{dx}+y=e^{2x}$ **(03)**

c) State Rolle's theorem. Verify it and find value of c **(04)**

for $f(x)=e^x(\sin x-\cos x),\ \forall x\in\left[\dfrac{\pi}{4},\dfrac{5\pi}{4}\right]$.

Q-7 Attempt all questions

a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix}2 & 1 & 1\\ 2 & 3 & 2\\ 3 & 3 & 4\end{bmatrix}$. **(07)**

b) If $A=\begin{bmatrix}1 & -1\\ 2 & 3\end{bmatrix}$ then find the characteristic polynomial of A and using it express $A^5-4A^4-7A^3+11A^2-A-10I$ as a linear polynomial in A **(07)**

Q-8 Attempt all questions

a) State and prove Cayley-Hamilton theorem. **(07)**

b) Solve: $p-\dfrac{1}{p}=\dfrac{x}{y}-\dfrac{y}{x}$ **(04)**

c) Solve: $y=p\left(x-p\right)+\dfrac{a}{p}$ **(03)**